Lecture 5

Equilibrium

Static deals primarily with the description of the force conditions necessary and sufficient to maintain the equilibrium of engineering structures.

When body is equilibrium, the resultant of all forces acting on it is zero. Thus, the resultant force R and the resultant couple m are both zero, and we have the equilibrium equations

$$\mathbf{R} = \Sigma \mathbf{F} = 0 \qquad \mathbf{M} = \Sigma \mathbf{M} = 0$$

These requirements are both necessary and sufficient conditions for equilibrium.

All physical bodies are three-dimensional, but we can treat many of them as two-dimensional when the forces to which they are subjected act in a single plane or can be projected onto a single plane. When this simplification is not possible, the problem must be treated as three

System Isolation And The Fee- body Diagram

Before we apply Eqs.3/1, we must define unambiguously the particular body or mechanical system to be analyzed and represent clearly and completely all forces actins oz the body. Omission of a force which acts on the body in question, or inclusion of a force which does not act on the body, will give erroneous results. A mechanical system is defined as a body or group of bodies which can be conceptually isolated from all other bodies. A system may be a single body or a combination of connected bodies. The bodies may be rigid or non rigid. The system may also be an identifiable fluid mass, either liquid or gas, or a combination of fluids and solids. In statics we study primarily forces which act on rigid bodies at rest, although we also study forces actins on fluids in equilibrium. Once we decide which body or combination of bodies to analyze, we then treat this body or combination as a single body isolated from all surrounding bodies. This isolation is accomplished by means of the free body diagram, which is a diagrammatic representation of the isolated system treated as a single body. The diagram shows all forces applied to the system by mechanical contact with other bodies, which are imagined to be removed If appreciable body forces are present. Such as gravitational or magnetic attraction, then these forces must also be shown on the free-body diagram of the isolated system.

Only after such a diagram has been carefully drawn should the equilibrium equations be written.

Because of its critical importance, we emphasize here that

the free-body diagram is the most important single step in the solution of problems in mechanics.

Before attempting to &aw a free-body diagram, we must recall the basic characteristics of force. These chrematistics were described in Art. 2/2, with primary attention focused on the vector properties of force. Forces can be applied either by direct physical contact or by remote action. Forces car be either internal or external to the system under consideration. Application of force is accompanied by reactive force, and both applied and reactive forces may be either concentrated or distributed. The principle of transmissibility permits the treatment of force as a sliding vector as far as its external effects on a rigid body are concerned.

We will now use these force characteristics to develop conceptual models of isolated mechanical systems. These models enable us to write the appropriate equations of equilibrium, which can then be analyzed.

Modeling the Action of Forces

Figure 1 shows the common types of force application on mechanical systems for analysis in two dimensions. Each example shows the force exerted on the body to be isolated, by the body to be removed. Newton's third law, which notes the existence of an equal and opposite reaction to every action, must be carefully observed. The force exerted on the body in question by a contacting or supporting member is always in the sense to oppose the movement of the isolated body which would occur if the contacting or supporting body were removed.

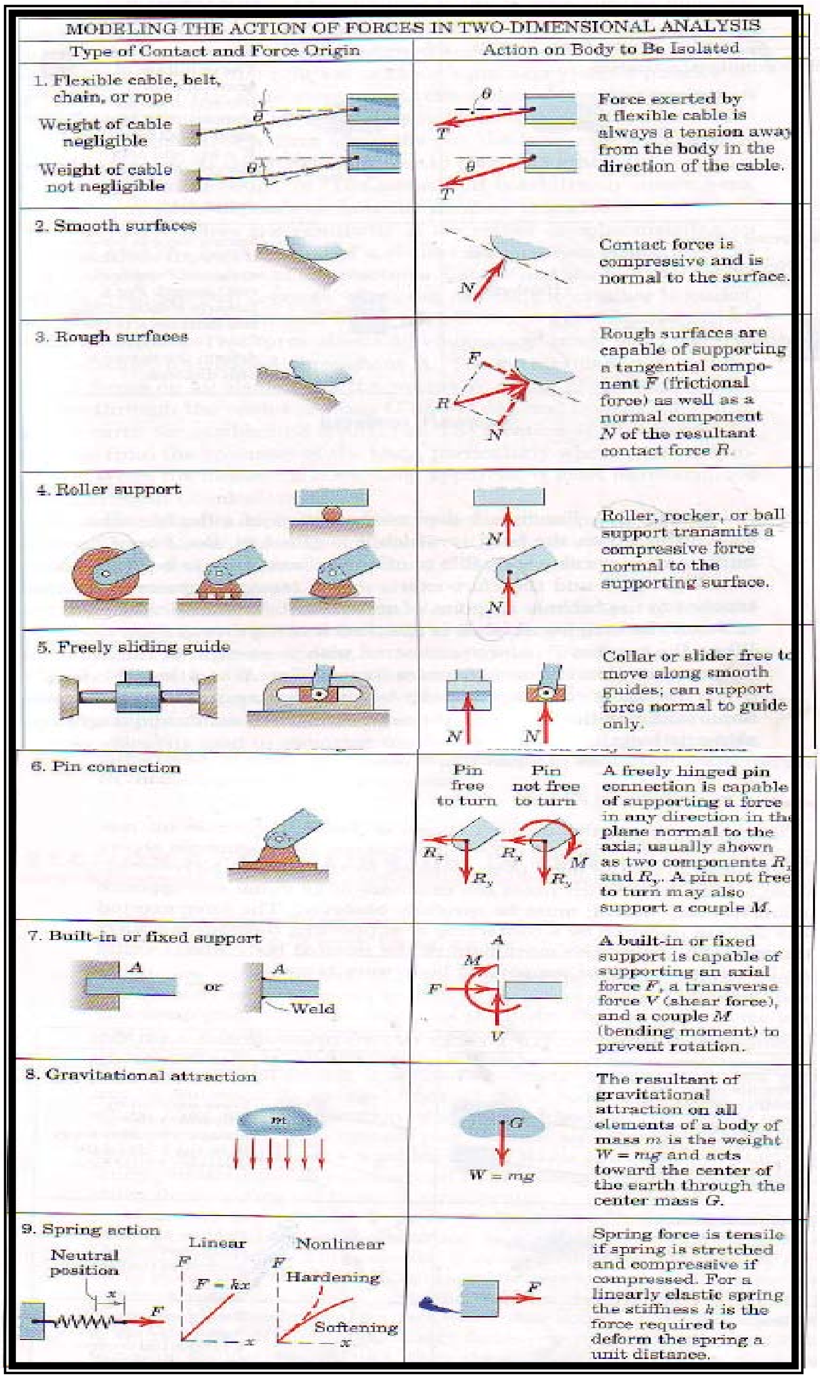
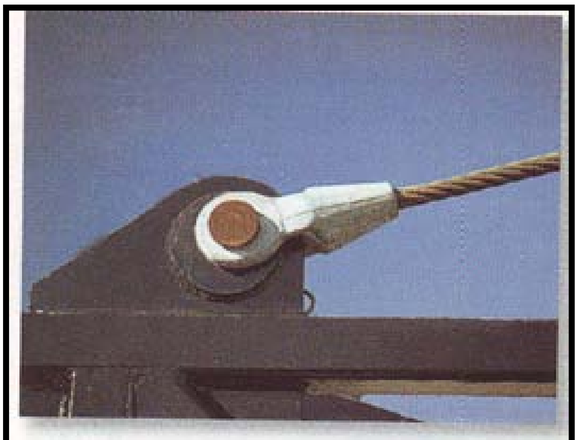


Figure1

Typical examples of actual supports that are referenced to Fig.1 are shown in the following sequence of photo



The cable exerts a force on the bracket in the direction of the cable.

This utility building is pin supported at the top of the column. (8)

The floor beams of this building are welded together and thus form fixed connections. (10)

In Fig. 1, Example 1 depicts the action of a flexible cable, belt, rope, or chain on the body to which it is attached. Because of its flexibility, a rope or cable is unable to offer any resistance to bending, shear, or compression and therefore exerts only a tension force in a direction tangent to the cable at its point of attachment. The force exerted by the cable on the body to which it is attached is always away from the body. When the tension T is large compared with the weight of the cable, we may assume that the cable forms a straight line. When the cable weight is not negligible compared with its tension, the sag of the cable becomes, important, and the tension in the cable changes direction and magnitude along its length.

When the smooth surfaces of two bodies are in contact. as in Example2 The force exerted by one on the other is normal to the tangent

to the surfaces and is compressive, Although no actual surfaces are perfectly smooth, we can assume this to be so for practical purposes in many instances.

When mating surfaces of contacting bodies are rough, as in Example3 , the force of contact is not necessarily normal to the tangent to the surfaces, but may be resolved into a tangential or frictional component F and a normal component N.

Example 4 illustrates a number of forms of mechanical support which effectively eliminate tangential friction forces. ln these cases the net reaction is normal to the supporting surface Example 5 shows the action of a smooth guide on the body it supports. There cannot be any resistance parallel to the guide

Example 6 illustrates the action of a pin connection. Such a connection can support force in any direction normal to the axis of the pin We usually represent this action in terms of two rectangular components. The correct sense of these components in a specific problem depends on how the member is loaded. when not otherwise initially known, the sense is arbitrarily assigned and the equilibrium equation are then written. If the solution of these equations yields a positive algebraic sign for the force component, the assigned sense is correct. A negative sign indicates the sense is opposite to that initially assigned.

If the joint is free to turn about the pin, the connection can support only the force R. If the joint is not free to turn, the connection can also support a resisting couple M. The sense of M is arbitrarily shown here, but the true sense depends on how the member is loaded.

Example 7 shows the resultants of the rather complex distribution of force over the cross section of a slender bar or beam at a built-in or fixed support. The sense of the reactions F and V and the bending couple M in a given problem depends of course, o how the member is loaded.

One of the most common forces is that due to gravitational attraction, Example 8. This force affects all elements of mass in a body and is, therefore. distributed throughout it. The resultant of the gravitational forces on all elements is the weight W = mg of the body, which passes through the center of mass G and is directed toward the center of the earth for earthbound structures The location of G is frequently obvious from the geometry of the body, particularly where there is symmetry. When the location is not readily apparent, it must be determined by experiment or calculations.

Similar remarks apply to the remote action of magnetic and electric forces. These forces of remote action have the same overall effect on a rigid body as forces of equal magnitude and direction applied by direct.

Example 9 illustrates the action of a linear elastic spring and of a nonlinear spring with either hardening or softening characteristics. The force exerted by a linear spring, in tension or compression, is given by F = kx, where k is the stiffness of the spring and x is its deformation measured from the neutral or unreformed position.

The representations in Fig. 1 are not free-body diagrams, but are merely elements used to construct free body diagrams. Study these nine conditions and identify them in the problem work so that you can draw the correct free-body diagrams.

Construction of Free-Body Diagrams

The full procedure for drawings a free-body diagram which isolates a body or system consists of the following steps

Step 1. Decide which system to isolate The system chosen should usually involve one or more of the desired unknown quantities.

Step 2. Next isolate the chosen system by drawing a diagram which represent its complete external boundary. This boundary defines the isolation of the system from all other attracting or contacting bodies, which are considered removed This step is often the most crucial of all. Make certain that you have completely isolated the system before proceeding with the next step.

Step 3. Identify{y all forces which act oz the isolated system as applied by the removed contacting and attracting bodies, and represent them in their proper positions on the diagram of the isolated system Make a systematic traverse of the entire boundary to identify all contact forces. Include body forces such as weights, where appreciable. Represent . all known forces by vector arrow, each with its! Proper magnitude, direction, and sense indicated. Each unknown force should be represented by a vector arrow with the unknown magnitude or direction indicated by symbol. if the sense of the vector is also unknown, you must arbitrarily assign a sense. The subsequent calculations with the equilibrium equations will yield a positive quantity if the incorrect sense was assumed and a negative quantity if the incorrect sense was assumed. it is necessary to be consistent with the assigned characteristics of unknown forces throughout all of the calculations. If you are consistent, the solution of the equilibrium equations will reveal the correct senses.

Step 4. Show the choice of coordinate axes directly on the diagram Pertinent dimensions may also be represented for convenience. Note, however., that the free-body diagram serves the purpose of focusing attention on the action of the external forces, and therefore the diagram should not be cluttered with excessive extraneous information. Clearly distinguish force arrows from arrows representing quantities other than forces. for this purpose a colored pencil may be used.

Completion of the foregoing four steps will produce a correct free-body diagram to use in applying the governing equations, both in statics and in dynamics. Be careful not to omit from the free-body diagram certain forces which may not appear at first glance to be needed in the calculations. lt is only through complete isolation and a systematic representation of all eternal forces that a reliable accounting of the effects of all applied and reactive forces can be made. very often a force which at first glance may not appear to influence a desired result does indeed have an influence. Thus. the only safe procedure is to include on the free-body diagram all forces whose magnitudes are not obviously negligible. The free-body method is extremely important in mechanics because it ensures an accurate definition of a mechanical system and focuses

attention on the exact meaning and application of the force laws of statics and dynamics. Review the foregoing four steps for constructing a free-body diagram while studying the sample free-body diagrams shown in Fig. 2 .

Examples of Free-Body Diagrams

Figure 2 gives four examples of mechanisms and structures together with their correct free-body diagrams. Dimensions and magnitudes are omitted for clarity. In each case we treat the entire system as a single body, so that the internal forces are not shown The characteristics of the various types of contact forces illustrated in Fig 1 are used in the four examples as they apply

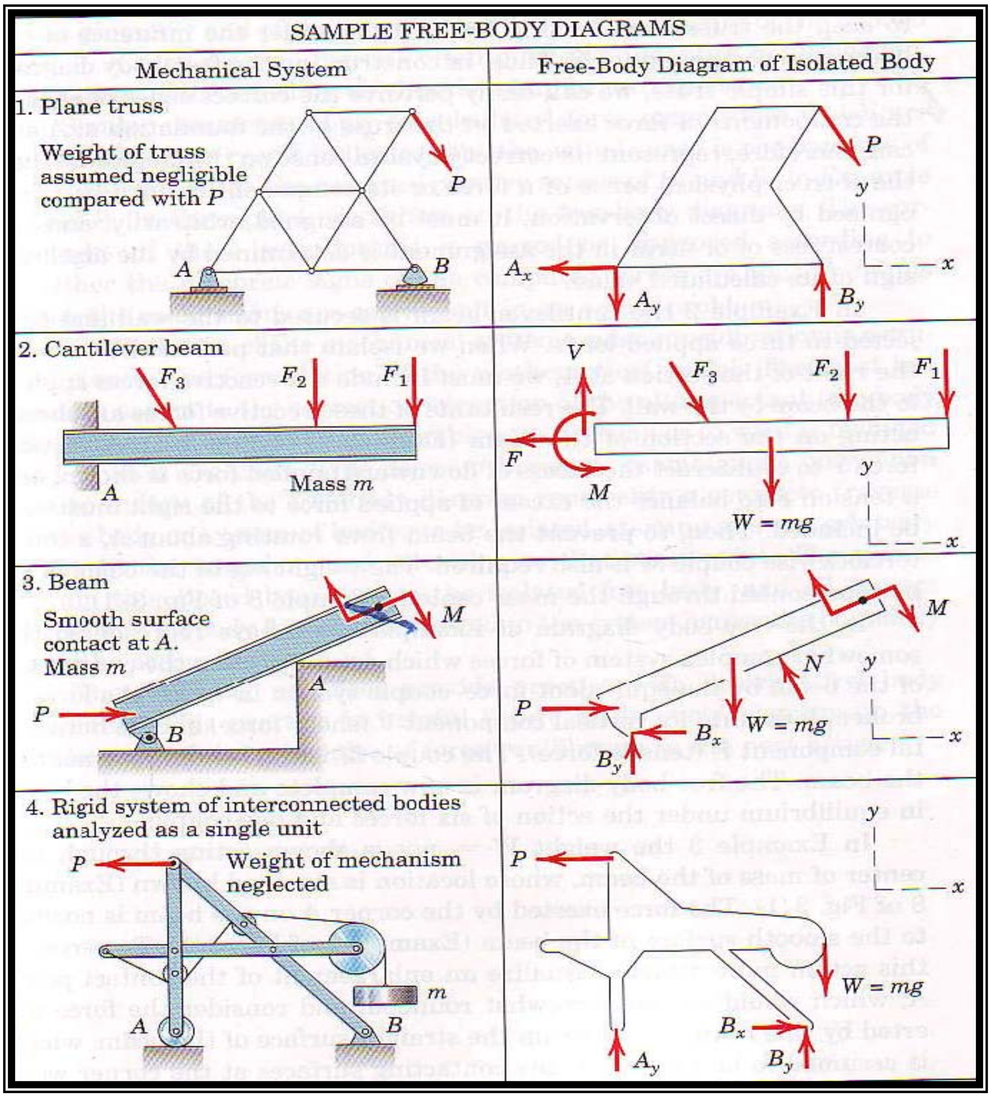
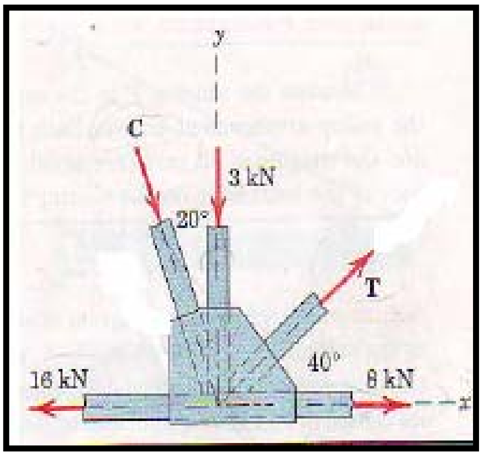


Figure 2

Examples

<u>Example 1</u>

Determine the magnitudes of the forces C and T, which, along with the other Forces shown, act on the bridge-truss joint.



Solution

The given sketch constitutes the free-body diagram of the isolated section of the joint in question and shows the five forces which ere in equilibrium

Solution 1 (scalar algebra): for the x-y axes as shown we have

Solution I (scalar algebra). For the x-y axes as shown we have

$[\Sigma F_x = 0]$ $\qquad 8 + T\cos 40^\circ + C\sin 20^\circ - 16 = 0$

$$0.766T + 0.342C = 8 \qquad (a)$$

$[\Sigma F_y = 0]$ $\qquad T\sin 40^\circ - C\cos 20^\circ - 3 = 0$

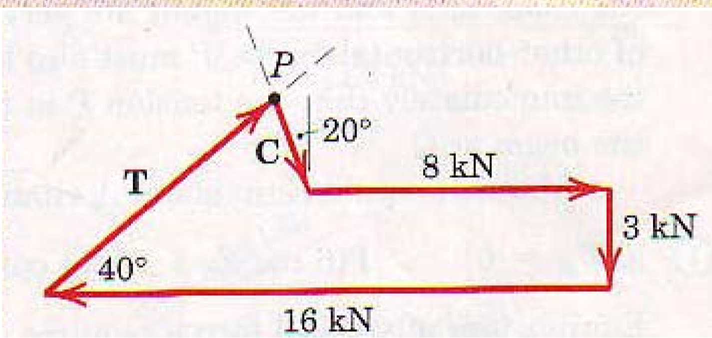
$$0.643T - 0.940C = 3 \qquad (b)$$

Simultaneous solution of Eqs. (a) and (b) produces

$$T = 9.09 \text{ kN} \qquad C = 3.03 \text{ kN} \qquad \textit{Ans.}$$

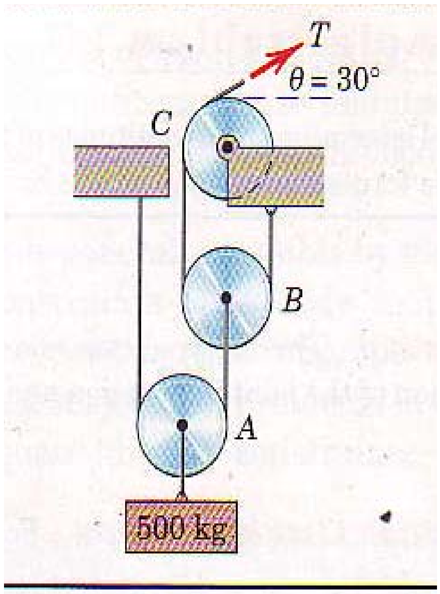
Solution IV (geometric). The polygon representing the zero vector sum of the five forces is shown. Equations (a) and (b) are seen immediately to give the projections of the vectors onto the x- and y-directions. Similarly, projections onto the x'- and y'-directions give the alternative equations in Solution II.

A graphical solution is easily obtained. The known vectors are laid off head-to-tail to some convenient scale, and the directions of **T** and **C** are then drawn to close the polygon. The resulting intersection at point P completes the solution, thus enabling us to measure the magnitudes of T and C directly from the drawing to whatever degree of accuracy we incorporate in the construction.



Example 2

Calculate the tension t in the cable which supports the 500-kg mass with the pulley arrangement shown. Each pulley is free to rotate about its bearing, and the weights of all parts are small compared with the load. Find the magnitude of the totl force on the bearing of pulley C.



Solution. The free-body diagram of each pulley is drawn in its relative position to the others. We begin with pulley *A*, which includes the only known force. With the unspecified pulley radius designated by *r*, the equilibrium of moments about its center *O* and the equilibrium of forces in the vertical direction require

$[\Sigma M_O = 0]$ $\quad T_1 r - T_2 r = 0 \quad T_1 = T_2$

$[\Sigma F_y = 0]$ $\quad T_1 + T_2 - 500(9.81) = 0 \quad 2T_1 = 500(9.81) \quad T_1 = T_2 = 2450\text{ N}$

From the example of pulley *A* we may write the equilibrium of forces on pulley *B* by inspection as

$$T_3 = T_4 = T_2/2 = 1226\text{ N}$$

For pulley *C* the angle $\theta = 30°$ in no way affects the moment of *T* about the center of the pulley, so that moment equilibrium requires

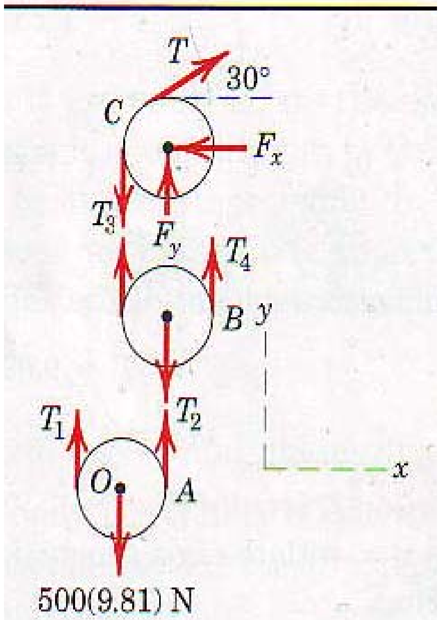
$$T = T_3 \quad \text{or} \quad T = 1226\text{ N} \qquad \textit{Ans.}$$

Equilibrium of the pulley in the *x*- and *y*-directions requires

$[\Sigma F_x = 0]$ $\quad 1226 \cos 30° - F_x = 0 \quad F_x = 1062\text{ N}$

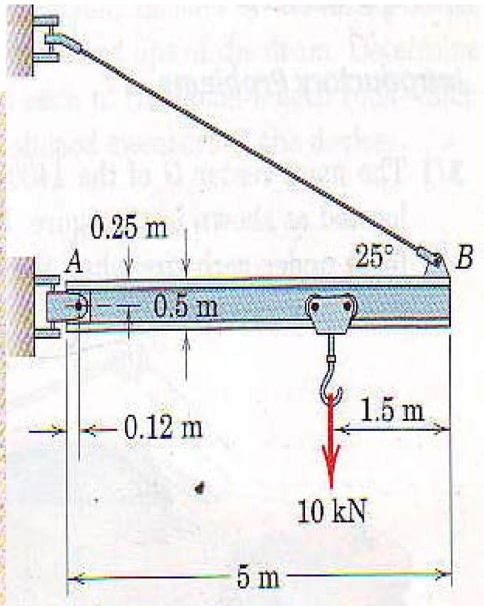
$[\Sigma F_y = 0]$ $\quad F_y + 1226 \sin 30° - 1226 = 0 \quad F_y = 613\text{ N}$

$[F = \sqrt{F_x^2 + F_y^2}]$ $\quad F = \sqrt{1062^2 + 613^2} = 1226\text{ N}$ *Ans.*



Example 3

Determine the magnitude T of the tension in supporting cable and the magnitude of the force on pin at A for the jip crane shown. The beam AB is a standarad 0.5-m I-beam with a mass of 95 kg per meter of length.



Algebraic solution. The system is symmetrical about the vertical x-y plane through the center of the beam, so the problem may be analyzed as the equilibrium of a coplanar force system. The free-body diagram of the beam is shown in the figure with the pin reaction at A represented in terms of its two rectangular components. The weight of the beam is $95(10^{-3})(5)9.81 = 4.66$ kN and acts through its center. Note that there are three unknowns A_x, A_y, and T which may be found from the three equations of equilibrium. We begin with a moment equation about A, which eliminates two of the three unknowns from the equation. In applying the moment equation about A, it is simpler to consider the moments of the x- and y-components of **T** than it is to compute the perpendicular distance from **T** to A. Hence, with the counterclockwise sense as positive we write

$[\Sigma M_A = 0]$ $\quad (T\cos 25°)0.25 + (T\sin 25°)(5 - 0.12) - 10(5 - 1.5 - 0.12) - 4.66(2.5 - 0.12) = 0$

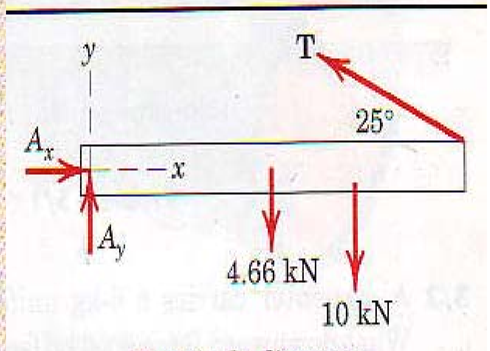
from which $\quad T = 19.61$ kN $\quad$ *Ans.*

Equating the sums of forces in the x- and y-directions to zero gives

$[\Sigma F_x = 0]$ $\quad A_x - 19.61\cos 25° = 0 \quad A_x = 17.77$ kN

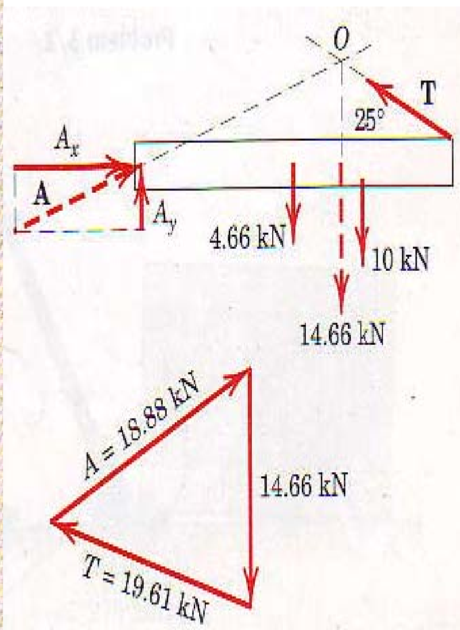
$[\Sigma F_y = 0]$ $\quad A_y + 19.61\sin 25° - 4.66 - 10 = 0 \quad A_y = 6.37$ kN

$[A = \sqrt{A_x^2 + A_y^2}]$ $\quad A = \sqrt{(17.77)^2 + (6.37)^2} = 18.88$ kN $\quad$ *Ans.*



Free-body diagram

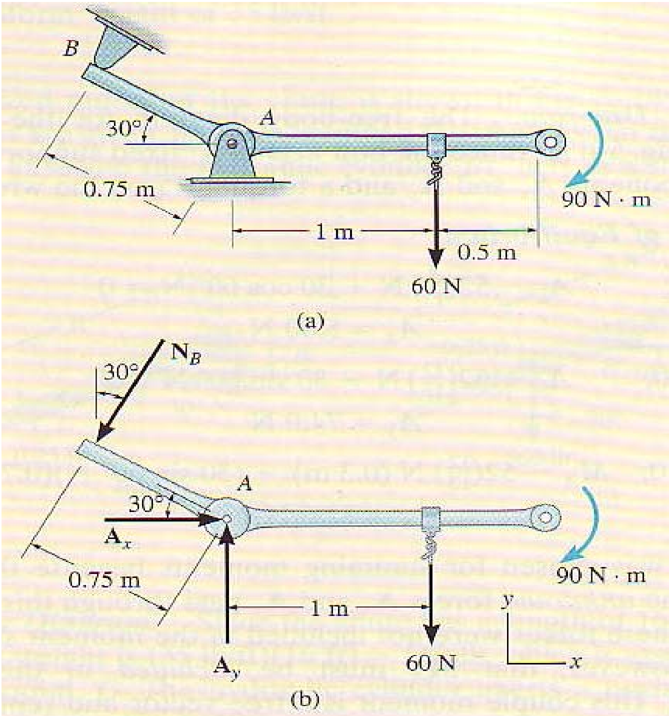
Graphical solution. The principle that three forces in equilibrium must be concurrent is utilized for a graphical solution by combining the two known vertical forces of 4.66 and 10 kN into a single 14.66-kN force, located as shown on the modified free-body diagram of the beam in the lower figure. The position of this resultant load may easily be determined graphically or algebraically. The intersection of the 14.66-kN force with the line of action of the unknown tension **T** defines the point of concurrency O through which the pin reaction **A** must pass. The unknown magnitudes of **T** and **A** may now be found by adding the forces head-to-tail to form the closed equilibrium polygon of forces, thus satisfying their zero vector sum. After the known vertical load is laid off to a convenient scale, as shown in the lower part of the figure, a line representing the given direction of the tension **T** is drawn through the tip of the 14.66-kN vector. Likewise a line representing the direction of the pin reaction **A**, determined from the concurrency established with the free-body diagram, is drawn through the tail of the 14.66-kN vector. The intersection of the lines representing vectors **T** and **A** establishes the magnitudes T and A necessary to make the vector sum of the forces equal to zero. These magnitudes are scaled from the diagram. The x- and y-components of **A** may be constructed on the force polygon if desired.



Graphical solution

Example 4

The link shown in Fig. a is pin-connected at A and rests against a smooth support at B. Compute the horizontal and vertical components of reaction at pin A.



Solution

Equations of Equilibrium. Summing moments about A, we obtain a direct solution for N_B,

$$\circlearrowleft + \Sigma M_A = 0; \quad -90 \text{ N}\cdot\text{m} - 60 \text{ N}(1 \text{ m}) + N_B(0.75 \text{ m}) = 0$$

$$N_B = 200 \text{ N}$$

Using this result,

$$\xrightarrow{+} \Sigma F_x = 0; \quad A_x - 200 \sin 30° \text{ N} = 0$$

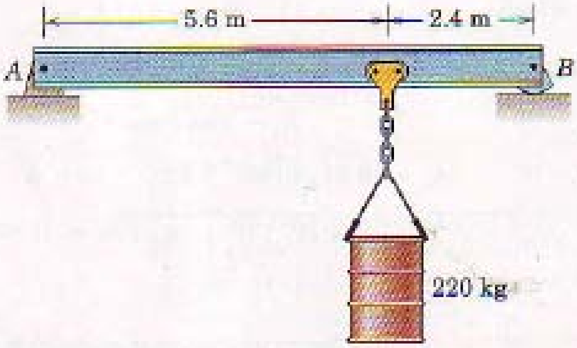
$$A_x = 100 \text{ N} \qquad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - 200 \cos 30° \text{ N} - 60 \text{ N} = 0$$

$$A_y = 233 \text{ N} \qquad \textit{Ans.}$$

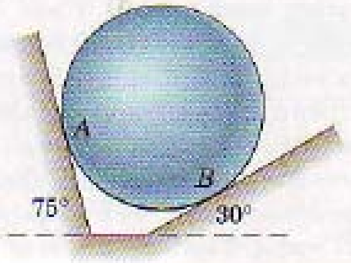
Problems

The 450-kg uniform I-beam supports the load shown. Determine the reactions at the supports.



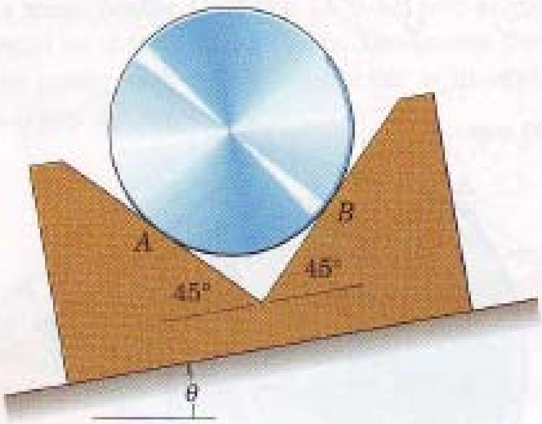
The 20-kg homogeneous smooth sphere rests on the two inclines as shown. Determine the contact forces at *A* and *B*.

Ans. $N_A = 101.6$ N, $N_B = 196.2$ N

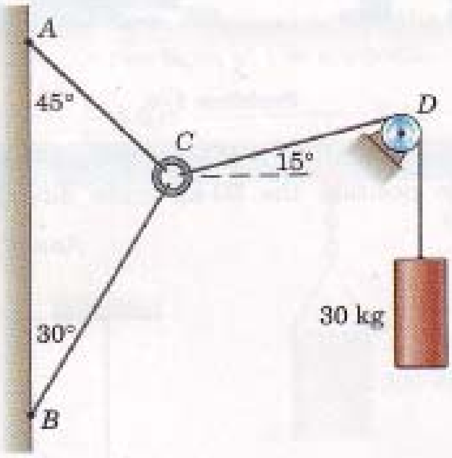


Find the angle of tilt θ with the horizontal so that the contact force at *B* will be one-half that at *A* for the smooth cylinder.

Ans. $\theta = 18.43°$

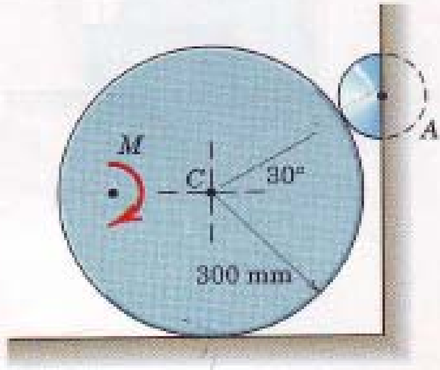


Three cables are joined at the junction ring *C*. Determine the tensions in cables *AC* and *BC* caused by the weight of the 30-kg cylinder.

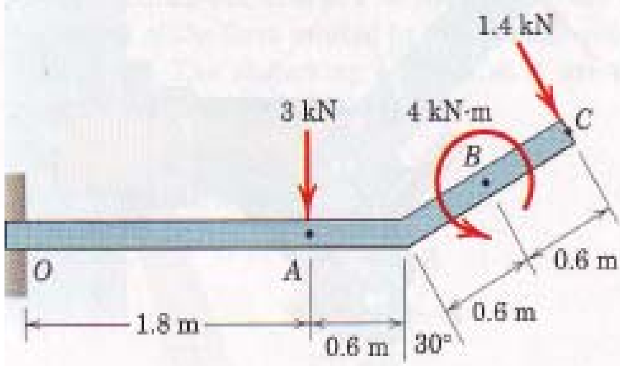


The 100-kg wheel rests on a rough surface and bears against the roller *A* when the couple *M* is applied. If $M = 60$ N·m and the wheel does not slip, compute the reaction on the roller *A*.

Ans. $F_A = 231$ N

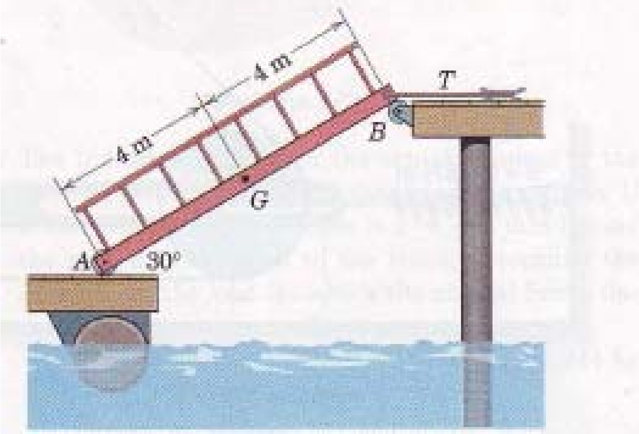


The uniform beam has a mass of 50 kg per meter of length. Compute the reactions at the support *O*. The force loads shown lie in a vertical plane.



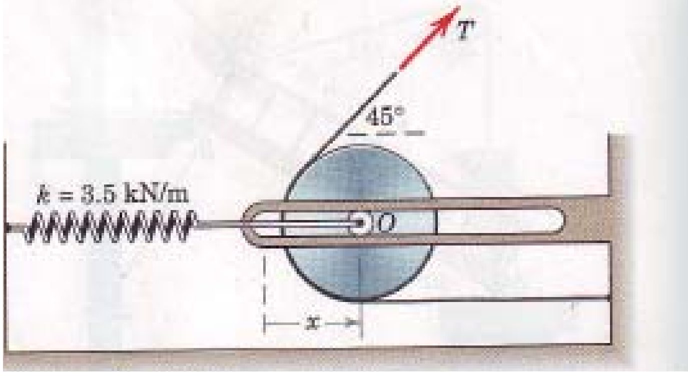
To accommodate the rise and fall of the tide, a walkway from a pier to a float is supported by two rollers as shown. If the mass center of the 300-kg walkway is at G, calculate the tension T in the horizontal cable which is attached to the cleat and find the force under the roller at A.

Ans. $T = 850$ N, $A = 1472$ N



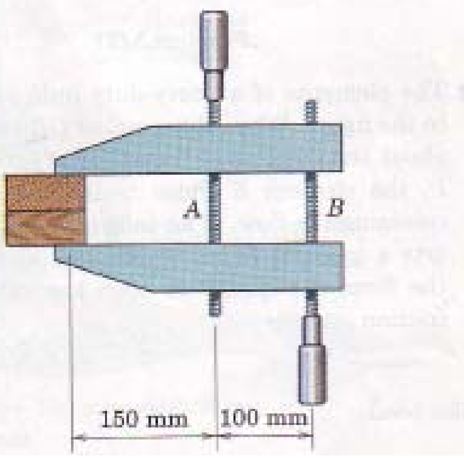
The spring of modulus $k = 3.5$ kN/m is stretched 10 mm when the disk center O is in the leftmost position $x = 0$. Determine the tension T required to position the disk center at $x = 150$ mm. At that position, what force N is exerted on the horizontal slotted guide? The mass of the disk is 3 kg.

Ans. $T = 328$ N, $N = 203$ N up

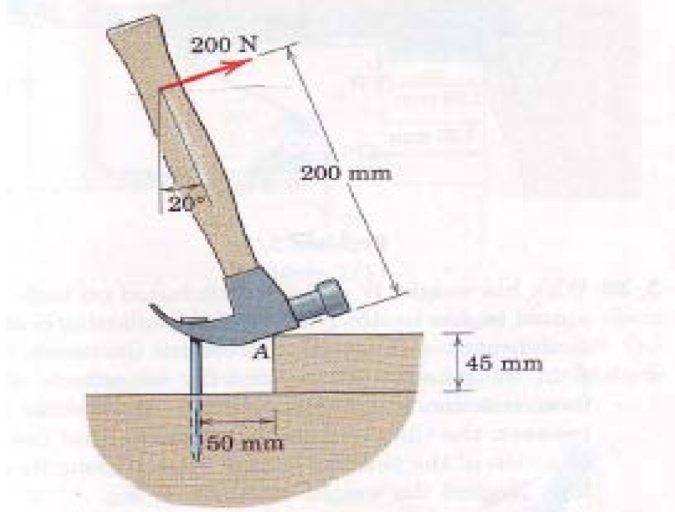


If the screw B of the wood clamp is tightened so that the two blocks are under a compression of 500 N, determine the force in screw A. (*Note:* The force supported by each screw may be taken in the direction of the screw.)

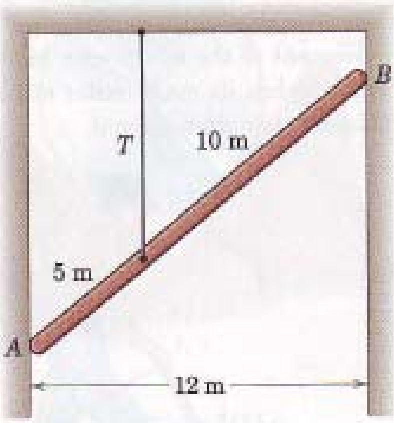
Ans. $A = 1250$ N



A block placed under the head of the claw hammer as shown greatly facilitates the extraction of the nail. If a 200-N pull on the handle is required to pull the nail, calculate the tension T in the nail and the magnitude A of the force exerted by the hammer head on the block. The contacting surfaces at A are sufficiently rough to prevent slipping.

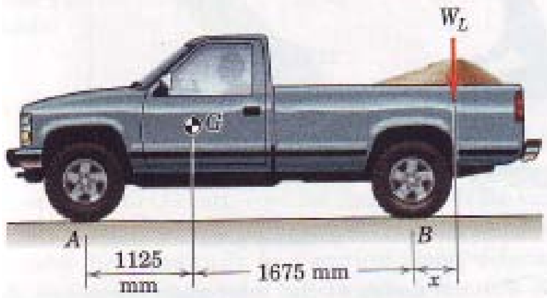


The uniform 15-m pole has a mass of 150 kg and is supported by its smooth ends against the vertical walls and by the tension T in the vertical cable. Compute the reactions at A and B.

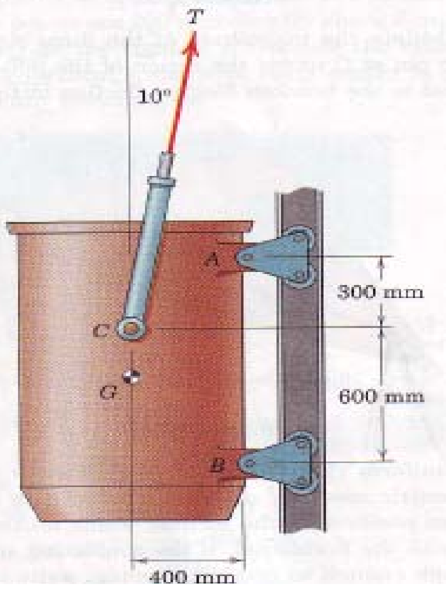


5 The indicated location of the center of mass of the 1600-kg pickup truck is for the unladen condition. If a load whose center of mass is x = 400 mm behind the rear axle is added to the truck, determine the mass m_L of the load for which the normal forces under the front and rear wheels are equal.

Ans. m_L = 244 kg

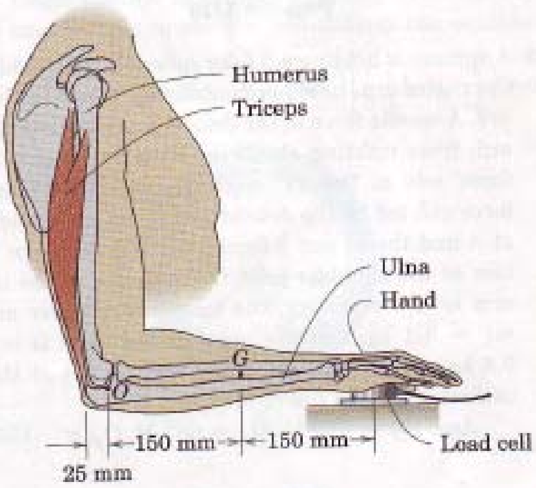


The concrete hopper and its load have a combir mass of 4 metric tons (1 metric ton equals 1000 l with mass center at G and is being elevated at c stant velocity along its vertical guide by the ca tension T. The design calls for two sets of guide r ers at A, one on each side of the hopper, and two s at B. Determine the force supported by each of two pins at A and by each of the two pins at B.

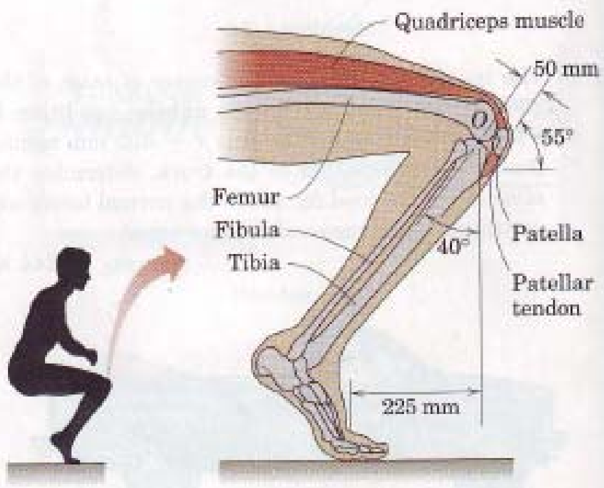


In a procedure to evaluate the strength of the triceps muscle, a person pushes down on a load cell with the palm of his hand as indicated in the figure. If the load-cell reading is 160 N, determine the vertical tensile force F generated by the triceps muscle. The mass of the lower arm is 1.5 kg with mass center at G. State any assumptions.

Ans. F = 1832 N



With his weight W equally distributed on both feet, a man begins to slowly rise from a squatting position as indicated in the figure. Determine the tensile force F in the patellar tendon and the magnitude of the force reaction at point O, which is the contact area between the tibia and the femur. Note that the line of action of the patellar tendon force is along its mid-line. Neglect the weight of the lower leg.



Determine the external reactions at A and F for the roof truss loaded as shown. The vertical loads represent the effect of the supported roofing materials, while the 400-N force represents a wind load.